RDF

Consider the graph $G$ describing holiday packages:

\[
\begin{align*}
_:b1 & \text{rdf:type } o:\text{Package} . \\
_:b1 & \text{destination } d:\text{Salvador} . \\
_:b1 & \text{accomodation } d:\text{PousadaDesArts} . \\
_:b1 & \text{activity } _:b2 . \\
_:b3 & \text{rdf:type } o:\text{Package} . \\
_:b3 & \text{destination } d:\text{Moskow} . \\
_:b3 & \text{accomodation } d:\text{Metropol} . \\
_:b3 & \text{activity } d:\text{VolgaCruise} . \\
_:b4 & \text{rdf:type } o:\text{Package} . \\
_:b4 & \text{destination } d:\text{Kobe} . \\
_:b4 & \text{accomodation } d:\text{ToyofukuRyokan} . \\
\end{align*}
\]

**WARNING:** the initial subject mentioned \( o:\text{type} \), instead of \( \text{rdf:type} \), this was a mistake.

1. Draw the graph $G$.
   The graph of Figure 1 corresponds to $G$.

2. Define an RDF-interpretation $\mathcal{I}$ of $G$.
   **WARNING:** actually this would be an interpretation of $G$'s vocabulary ($V(G)$). For the next question, I need a model.
   \[
   \mathcal{I} = \langle I_R, I_P, I_{EXT}, \iota \rangle
   \]
   such that:
   \[
   I_R \supseteq I_P \cup \{B, C, D\}
   \]
   \[
   I_P \supseteq \{\iota(\text{rdf:type}), \iota(\text{o:destination}), \iota(\text{o:accomodation}), \iota(\text{o:activity})\}
   \]
   \[
   I_{EXT}(\iota(\text{rdf:type})) \supseteq \{ \langle B, \iota(\text{o:Package}) \rangle, \langle C, \iota(\text{o:Swimming}) \rangle, \langle D, \iota(\text{o:Cruise}) \rangle \}
   \]
   \[
   I_{EXT}(\iota(\text{destination})) \supseteq \{ \langle B, \iota(\text{d:Salvador}) \rangle, \langle B, \iota(\text{d:Moskow}) \rangle, \langle B, \iota(\text{d:Kobe}) \rangle \}
   \]
   \[
   I_{EXT}(\iota(\text{accomodation})) \supseteq \{ \langle B, \iota(\text{d:PousadaDesArts}) \rangle, \langle B, \iota(\text{d:Metropol}) \rangle, \langle B, \iota(\text{d:ToyofukuRyokan}) \rangle \}
   \]
   \[
   I_{EXT}(\iota(\text{activity})) \supseteq \{ \langle B, C \rangle, \langle B, \iota(\text{d:VolgaCruise}) \rangle, \langle B, D \rangle \}
   \]
   It is possible to replace $\iota(\ldots)$ by $a, b, \ldots$ if it makes you more comfortable. This interpretation is a bit peculiar as it interprets all packages as the same with three destinations, but nothing prohibits this.

3. Given the following graph $H$:
Figure 1: RDF graph $G$.

Does your interpretation satisfies $H$ (said otherwise, is $\mathcal{I}$ a model of $H$)?

Yes, $\mathcal{I}$ a model of $H$ as it is possible to find an extension $\iota'$ of $\iota$ to $\{_:x, _:act, _:acc\}$ satisfying all triples of $H$. This is the case, for instance if one takes: $\iota' = \iota \cup \{\langle _:x, B \rangle, \langle _:act, C \rangle, \langle _:acc, d:PousadaDesArts \rangle\}$.

4. Does $G \models H$? Show it.

Any model of $G$ is indeed a model of $H$. For any model $m = \langle I_R, I_P, I_{EXT}, \iota \rangle$ of $G$, $\iota$ can be extended into $\iota'$ such that:

\[
\langle \iota'(_:b1), \iota(o:Package) \rangle \in I_{EXT}(\iota(rdf:type))
\]
\[
\langle \iota'(_:b1), \iota(d:PousadaDesArts) \rangle \in I_{EXT}(\iota(o:accomodation))
\]
\[
\langle \iota'(_:b1), \iota'(_:b2) \rangle \in I_{EXT}(\iota(o:activity))
\]

so it is possible to define the extension $\iota''$ of $\iota'$ to $\{_:x, _:act, _:acc\}$ such that: $\iota''(_:x) = \iota'(_:b1)$, $\iota''(_:act) = \iota'(_:b2)$, and $\iota''(_:acc) = \iota(d:PousadaDesArts)$. $\iota''$ is an extension of $\iota$ and it satisfies all triples of $H$, hence, $m$ is a model of $H$. This can also be achieved by showing that there is an RDF-homomorphism from $H$ to $G$ or that an instance of $H$ is a subgraph of $G$.

5. Given the following graph $K$:

\[
_:y rdf:type o:Package .
\]
\[
_:y o:accomodation _:acc .
\]
\[
_:acc rdf:type o:Local .
\]
\[
_:y o:activity _:act .
\]
\[
_:act rdf:type o:Sport .
\]
Does $G \models K$? Tell why.

No, because there is no reference to $o:Sport$ in the graph $G$, hence it is impossible to find an RDF-homomorphism from an instance of $K$ to a subgraph of $G$ as it would need to map the node labelled by $o:Sport$ to a node with the same label (see also answer to Question 7).

**RDFS and OWL interpretation**

Consider the ontology $O$ made of the following statements:

- $o:accomodation$ rdfs:range o:Accomodation.
- $o:Local$ rdfs:subClassOf o:Accomodation.
- $o:Pousada$ rdfs:subClassOf o:Local.
- $o:Ryokan$ rdfs:subClassOf o:Local.
- $o:GrandHotel$ rdfs:subClassOf Accomodation.
- $o:activity$ rdfs:range o:Activity.
- $o:Sport$ rdfs:subClassOf o:Activity.
- $o:Swimming$ rdfs:subClassOf o:Sport.
- $o:SwordFighting$ rdfs:subClassOf o:Sport.
- $o:Visit$ rdfs:subClassOf o:Activity.
- $o:Cruising$ rdfs:subClassOf o:Visit.

6. Does $G \models_{RDFS} o:Package$ rdf:type rdfs:Class?

   $O \models_{RDFS} o:Package$ rdf:type rdfs:Class

   Because $o:Package$ is the rdf:type of items, this entails that it is a class. Indeed, by the RDF semantics (1), $\langle \iota(o:Package), \iota(o:Package) \rangle \in I_{EXT}(\iota(o:Package))$; but all axiomatic triples are satisfied (2c) and in particular $\langle \iota(o:Package), \iota(rdfs:Class) \rangle \in I_{EXT}(\iota(o:Package))$. Since, this is true for all models of $G$, this means that all these models satisfy $\langle o:Package$ rdf:type rdfs:Class $\rangle$.

   $O \not\models_{RDFS} o:Package$ rdf:type rdfs:Class

   Because, since there is no mention of $o:Package$ in $O$, this does not allow to entail anything about it. More precisely, there is no constraint in $O$ preventing that $\iota(o:Package) \in I_R \setminus$ Class.

7. Does $O \cup G \models_{RDF} K$? $O \cup G \models_{RDFS} K$? Explain why.

   $O \cup G \not\models_{RDF} K$

   For this to be satisfied, it would be necessary that an instance of $K$ be a subgraph of $O \cup G$. This would necessitate a triple whose predicate is rdf:type and whose object be $o:Sport$. But no such triple exist either in $O$ or in $G$.

   $O \cup G \models_{RDFS} K$

   Indeed, if one computes the (partial) closure of $O \cup G$, then it contains $\langle :b5, rdf:type, o:Sport \rangle$ (and $d:ToyofukuRyokan, rdf:type, o:Local \rangle$ by rule [RDFS11]) because, $G$ contains $\langle :b5, rdf:type, o:SwordFighting \rangle$ (and $d:ToyofukuRyokan, rdf:type, o:Ryokan \rangle$) and $O$ contains $\langle o:Ryokan, rdfs:subClassOf, o:Local \rangle$ (and $o:SwordFighting, rdfs:subClassOf, o:Sport \rangle$). Thus, it is possible to define an RDF-homomorphism $h : K \to cl(O \cup G)$ such that $h(\_y) = :b4$, $h(\_acc) = d:ToyofukuRyokan$, $h(\_act) = :b5$ and $h(K) \in cl(O \cup G)$. $h$ is indeed an homomorphism as it preserves the graph structure of $K$.

8. Given the OWL axiom (making the OWL ontology $O'$):

   $o:TonicPackage \equiv o:Package$

   $\exists o:accomodation(o:Local \geq_1 o:swimmingPool)$

   $\exists o:activity.o:Sport$
Give the OWL interpretation of TonicPackage ($E_C(o:TonicPackage)$).

WARNING: The initial exam was not using $\geq 1$ but an equivalent formulation. It will, of course, be corrected accordingly.

$$E_C(o:TonicPackage) = E_C(o:Package)$$
\[
\cap \exists o:accomodation.(o:Local \cap \geq 1 o:swimmingPool)
\cap \exists o:activity.o:Sport)
\]

$$= E_C(o:Package)$$
\[
\cap E_C(\exists o:accomodation.(o:Local \cap \geq 1 o:swimmingPool))
\cap E_C(\exists o:activity.o:Sport))
\]

$$= E_C(o:Package)$$
\[
\cap \{ x|(x,y) \in E_R(o:accomodation) \land y \in E_C(o:Local \cap \geq 1 o:swimmingPool))
\cap \{ x|(x,y) \in E_R(o:activity) \land y \in E_C(o:Sport})
\]

$$= E_C(o:Package)$$
\[
\cap \{ x|(x,y) \in E_R(o:accomodation) \land y \in E_C(o:Local) \cap \{ z|\#\{ z,t \in E_R(o:swimmingPool)) \geq 1\})
\cap \{ x|(x,y) \in E_R(o:activity) \land y \in E_C(o:Sport})
\]

9. Does $O \cup O' \cup G \models_{\text{OWL}} _{-} \text{b1 rdf:type o:TonicPackage}$? Tell why.

WARNING: The initial exam was referring to _-b1 instead of _-b, so answers taking this into account are accepted.

The definition of o:TonicPackage constrains its instances have an accommodation that has at least one swimming pool. However, neither O nor G refer to o:swimmingPool, hence there can be models of $O \cup O' \cup G$ in which $E_C(o:swimmingPool) = \emptyset$ and thus $E_C(o:TonicPackage) = \emptyset$. Obviously, such models do not satisfy _-b1 rdf:type o:TonicPackage. Hence this statement is not a consequence.

10. Can you express a SPARQL query returning all o:TonicPackage as defined in the OWL axiom of question 8?

```
SELECT ?p
WHERE {
  ?acc rdf:type o:Local .
  ?act rdf:type o:Sport .
}
```