4 OWL 2 qualified cardinality restrictions

OWL 2 introduced qualified cardinality restrictions (owl:qualifiedCardinality, owl:maxQualifiedCardinality, and owl:minQualifiedCardinality, whose interpretation is obtained by extending the $E_C$ function of Definition 19:

$$E_C\left(\text{restriction}(p, \text{minQualifiedCardinality}(n, C))\right) = \{x \in O; |\{(x, y) \in E_R(p); y \in E_C(C)\}| \geq n\}$$

$$E_C\left(\text{restriction}(p, \text{maxQualifiedCardinality}(n, C))\right) = \{x \in O; |\{(x, y) \in E_R(p); y \in E_C(C)\}| \leq n\}$$

$$E_C\left(\text{restriction}(p, \text{qualifiedCardinality}(n, C))\right) = \{x \in O; |\{(x, y) \in E_R(p); y \in E_C(C)\}| = n\}$$

Consider the following expressions (in OWL 2):

```xml
<owl:Class rdf:about="#SmallTeam">
  <rdfs:subClassOf>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#member"/>
      <owl:maxCardinality rdf:datatype="&xsl;integer">5</owl:maxCardinality>
    </owl:Restriction>
  </rdfs:subClassOf>
</owl:Class>

<owl:Class rdf:about="#ModernTeam2">
  <rdfs:subClassOf rdf:resource="#SmallTeam"/>
  <rdfs:subClassOf>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#member"/>
      <owl:minQualifiedCardinality rdf:datatype="&xsl;integer">4</owl:minQualifiedCardinality>
      <owl:onClass rdf:resource="#Woman"/>
    </owl:Restriction>
  </rdfs:subClassOf>
</owl:Class>
```

1. Draw the graph corresponding to this set of triples.

2. Express it in OWL/XML.
3. Explain the meaning of this graph (paraphrase it in English)

“A modern team is a small team with at least 4 female team members; A small team is a team with at most 5 members;”

4. What would happen if we exchange the 5 and the 4?

Then the paraphrase would be that “A modern team is a small team with at least 5 female team members; A small team is a team with at most 4 members;” hence, there could not exists any modern team.

Consider the following statements:

ex:MyTeam rdf:type ex:ModernTeam2 .
ex:Kay ex:member ex:MyTeam .
ex:Kay rdf:type ex:Man .
ex:Jo ex:member ex:MyTeam .

5. If one queries this graph with `SELECT ?x WHERE ?x ex:member ex:MyTeam . ?x rdf:type ex:Woman .`, what would be the answer?

The answer would be empty because no team member is explicitly declared as a `ex:Woman`. Hence there is no way to project the triple pattern `?x rdf:type ex:Woman .` into a triple of the graph.

Even if queries were interpreted with respect to ontologies, this would not be sufficient to entail that `ex:Jo` is a `ex:Woman` because it is not excluded that `ex:Jo` and `ex:Kay` are different identifiers for the same resource. Even then, it is not prevented by the ontology that `ex:Woman` and `ex:Man` are disjoint, hence `ex:Kay` may be both and `ex:Jo` not be a `ex:Woman`. 

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Figure 1: The RDF graph of the OWL 2 ontology.
6. What would be necessary for **ex:Jo** to be an answer?

The simplest solution would be to have **ex:Jo** `rdf:type ex:Woman` in the graph.
If queries are interpreted modulo ontologies, it would be sufficient to have:

- **ex:Man** `owl:disjointFrom ex:Woman`.
- **ex:Jo** `owl:differentFrom ex:Kay`.

in the graph.

5 From OWL 2 to OWL 1 and back

1. How is it possible to rewrite `qualifiedCardinality` in function of the minimal and maximal qualified cardinality restrictions? Explain it with the semantics.

   `qualifiedCardinality` can be rewritten with respect to minimal and maximal qualified cardinality restrictions by replacing each of its occurrences by the conjunction of the others:

   \[
   \text{restriction}(p, \text{qualifiedCardinality}(n, C)) \equiv \text{restriction}(p, \text{minQualifiedCardinality}(n, C)) \land \text{restriction}(p, \text{maxQualifiedCardinality}(n, C))
   \]

   where \( \land \) represent class conjunction (`owl:intersectionOf`). Indeed,

   \[
   E_C(\text{restriction}(p, \text{qualifiedCardinality}(n, C))) = \{ x \in O ; |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| = n \}
   = \{ x \in O ; n \leq |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| \leq n \}
   = \{ x \in O ; |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| \leq n \}
   \land |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| \geq n \}
   = \{ x \in O ; |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| \leq n \}
   \land |\{(x, y) \in E_R(p) ; y \in E_C(C)\}| \geq n \}
   = E_C(\text{restriction}(p, \text{maxQualifiedCardinality}(n, C))) \land E_C(\text{restriction}(p, \text{minQualifiedCardinality}(n, C)))
   = E_C(\text{restriction}(p, \text{maxQualifiedCardinality}(n, C)))
   \land E_C(\text{restriction}(p, \text{minQualifiedCardinality}(n, C)))
   \]

2. Is it possible to express `minCardinality`, `maxCardinality`, `cardinality`, `someValuesFrom` with these new qualified cardinality restrictions? Explain how.

   It is possible:

   \[
   \text{restriction}(p, \text{minCardinality}(n)) \equiv \text{restriction}(p, \text{minQualifiedCardinality}(n, \text{Thing}))
   \]

   \[
   \text{restriction}(p, \text{maxCardinality}(n)) \equiv \text{restriction}(p, \text{maxQualifiedCardinality}(n, \text{Thing}))
   \]

   \[
   \text{restriction}(p, \text{cardinality}(n)) \equiv \text{restriction}(p, \text{qualifiedCardinality}(n, \text{Thing}))
   \]

   \[
   \text{restriction}(p, \text{someValuesFrom}(C)) \equiv \text{restriction}(p, \text{minQualifiedCardinality}(1, C))
   \]

Consider, in addition to the previous RDF graphs, the following statements (expressed in OWL 1):

- **ex:womanmember** `owl:subPropertyOf ex:member`.
- **ex:womanmember** `rdfs:range ex:Woman`.
- **ex:ModernTeam1** `rdfs:subClassOf ex:SmallTeam`.
- **ex:ModernTeam1** `rdfs:subClassOf __:b`.
- __:b `rdf:type owl:Restriction`.
- __:b `owl:onProperty ex:womanmember`.
- __:b `owl:minCardinality 4`.
3. Does ex:ModernTeam1 subsume ex:ModernTeam2 or the other way around? Justify.

It is clear that any ex:ModernTeam1 is a ex:ModernTeam2 because the ex:womanmember are ex:members who are ex:Woman, so having 4 of them will satisfy the required constraint in ex:ModernTeam2. Hence, ex:ModernTeam2 subsumes ex:ModernTeam1.

The other way around is less clear a priori. However, when one interprets a statement like:

<owl:ObjectProperty rdf:about="#womanmember">
  <owl:subPropertyOf rdf:about="#member" />
  <rdfs:range rdf:about="#Woman"/>
</owl:ObjectProperty>

in OWL 1 semantics, this is interpreted definitionally, i.e., ex:womanmember is equivalent to those ex:member who are ex:Woman. I.e.,

\[
E_C(\text{restriction}(\text{ex:womanmember, minCardinality}(4)))
= \{x \in O; |\{(x, y) \in E_R(\text{ex:womanmember})| \geq 4\}
= \{x \in O; |\{(x, y) \in E_R(\text{ex:member}) \cap \text{range}(\text{ex:Woman})| \geq 4\}
= \{x \in O; |\{(x, y) \in E_R(\text{ex:member}) \cap E_R(\text{range}(\text{ex:Woman}))| \geq 4\}
= \{x \in O; |\{(x, y) \in E_R(\text{ex:member}) \cap \text{E}(\text{ex:Woman}))| \geq 4\}
= \{x \in O; |\{(x, y) \in E_R(\text{ex:member}); y \in E(\text{ex:Woman}))| \geq 4\}
= E_C(\text{restriction}(\text{ex:member, minQualifiedCardinality}(4, \text{ex:Woman})))
\]

Hence the two expressions are equivalent and ex:ModernTeam1 subsumes ex:ModernTeam2.

4. Does this suggest that it is also possible to express qualified cardinality constraints in OWL 1? Explain.

This indeed suggests that, by adding property definitions, such as ex:womanmember, it is possible to transform any OWL 1 ontology with qualified cardinality restrictions into an equivalent plain OWL 1 ontology, i.e., without qualified cardinality restrictions.

5. Does qualified cardinality restrictions provide additional expressivity to OWL 1?

Hence, no. Qualified cardinality restrictions are only syntactic sugar.