M2R Exam – Semantic web: from XML to OWL
Semantic web part

Duration : 1h30
Any document allowed – no communication device allowed

January 2013

Note: Please, carefully read all the questions before answering.

4 OWL 2 qualified cardinality restrictions

OWL 2 introduced qualified cardinality restrictions (`owl:qualifiedCardinality`, `owl:maxQualifiedCardinality`, and `owl:minQualifiedCardinality`, whose interpretation is obtained by extending the E_C function of Definition 19:

$$E_C(\texttt{restriction}(p, \texttt{minQualifiedCardinality}(n, C))) = \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \geq n\}$$
$$E_C(\texttt{restriction}(p, \texttt{maxQualifiedCardinality}(n, C))) = \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \leq n\}$$
$$E_C(\texttt{restriction}(p, \texttt{qualifiedCardinality}(n, C))) = \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| = n\}$$

Consider the following expressions (in OWL 2):

```
ex:SmallTeam rdfs:subClassOf _:a .
_:a rdf:type owl:Restriction .
_:a owl:onProperty ex:member .
_:a owl:maxCardinality 5 .
ex:ModernTeam2 rdfs:subClassOf ex:SmallTeam .
ex:ModernTeam2 rdfs:subClassOf _:b .
_:b rdf:type owl:Restriction .
_:b owl:onProperty ex:member .
_:b owl:minQualifiedCardinality 4 .
_:b owl:onClass ex:Woman .
```

1. Draw the graph corresponding to this set of triples.
2. Express it in OWL/XML.

```
<owl:Class rdf:about="#SmallTeam">
  <rdfs:subClassOf>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#member"/>
      <owl:maxCardinality rdf:datatype="&xsl;integer">5</owl:maxCardinality>
    </owl:Restriction>
  </rdfs:subClassOf>
</owl:Class>

<owl:Class rdf:about="#ModernTeam2">
  <rdfs:subClassOf rdf:resource="#SmallTeam"/>
```

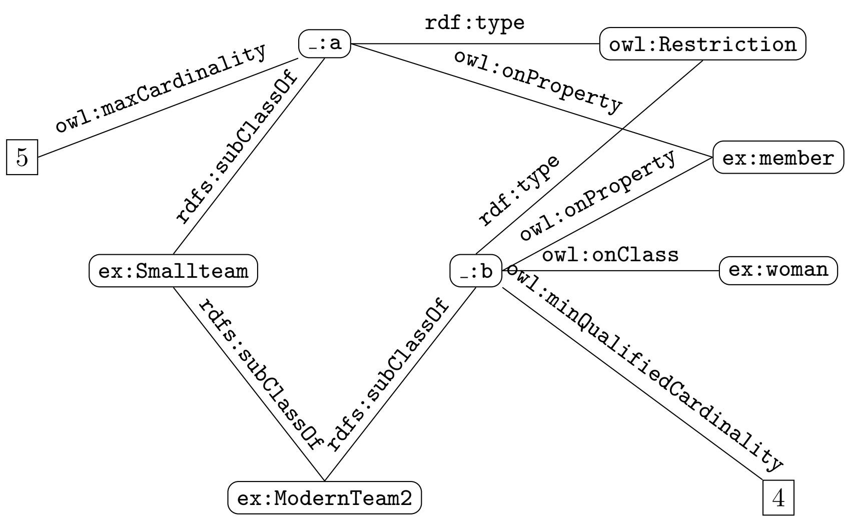

Figure 1: The RDF graph of the OWL 2 ontology.

```
  <rdfs:subClassOf>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#member"/>
      <owl:minQualifiedCardinality rdf:datatype="&xsl;integer">4</owl:maxCardinality>
      <owl:onClass rdf:resource="#Woman"/>
    </owl:Restriction>
  </rdfs:subClassOf>
</owl:Class>
```

3. Explain the meaning of this graph (paraphrase it in English)

   "A modern team is a small team with at least 4 female team members; A small team is a team with at most 5 members;"

4. What would happen if we exchange the 5 and the 4?

   Then the paraphrase would be that "A modern team is a small team with at least 5 female team members; A small team is a team with at most 4 members;" hence, there could not exists any modern team.

Consider the following statements:

```
ex:MyTeam rdf:type ex:ModernTeam2 .
ex:Kay ex:member ex:MyTeam .
ex:Kay rdf:type ex:Man .
ex:Jo ex:member ex:MyTeam .
```

5. If one queries this graph with `SELECT ?x WHERE ?x ex:member ex:MyTeam . ?x rdf:type ex:Woman .`, what would be the answer?

   The answer would be empty because no team member is explicitely declared as a `ex:Woman`. Hence there is no way to project the triple pattern `?x rdf:type ex:Woman .` into a triple of the graph.

   Even if queries were interpreted with respect to ontologies, this would not be sufficient to entail that `ex:Jo` is a `ex:Woman` because it is not excluded that `ex:Jo` and `ex:Kay` are different identifiers for the same resource. Even then, it is not prevented by the ontology that `ex:Woman` and `ex:Man` are disjoint, hence `ex:Kay` may be both and `ex:Jo` not be a `ex:Woman`.

6. What would be necessary for `ex:Jo` to be an answer?

   The simplest solution would be to have `ex:Jo rdf:type ex:Woman .` in the graph.

   If queries are interpreted modulo ontologies, it would be sufficient to have:

```
ex:Man owl:disjointFrom ex:Woman .
ex:Jo owl:differentFrom ex:Kay .
```

   in the graph.

5 From OWL 2 to OWL 1 and back

1. How is it possible to rewrite `qualifiedCardinality` in function of the minimal and maximal qualified cardinality restrictions? Explain it with the semantics.

   `qualifiedCardinality` can be rewritten with respect to minimal and maximal qualified cardinality restrictions by replacing each of its occurences by the conjunction of the others:

$$\begin{aligned}\texttt{restriction}(p, \texttt{qualifiedCardinality}(n, C))) &\equiv \texttt{restriction}(p, \texttt{minQualifiedCardinality}(n, C))) \\ &\sqcap \texttt{restriction}(p, \texttt{maxQualifiedCardinality}(n, C)))\end{aligned}$$

   where $\sqcap$ represent class conjunction (`owl:intersectionOf`). Indeed,

$$\begin{aligned}E_C(\texttt{restriction}(p, \texttt{qualifiedCardinality}(n, C))) &= \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| = n\} \\ &= \{x \in O; n \leq |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \leq n\} \\ &= \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \leq n \\ &\quad \wedge |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \geq n\} \\ &= \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \leq n\} \\ &\cap \{x \in O; |\{\langle x, y\rangle \in E_R(p); y \in E_C(C)\}| \geq n\} \\ &= E_C(\texttt{restriction}(p, \texttt{maxQualifiedCardinality}(n, C))) \\ &\cap E_C(\texttt{restriction}(p, \texttt{minQualifiedCardinality}(n, C))) \\ &= E_C(\texttt{restriction}(p, \texttt{maxQualifiedCardinality}(n, C)) \\ &\quad \sqcap \texttt{restriction}(p, \texttt{minQualifiedCardinality}(n, C)))\end{aligned}$$

2. Is it possible to express `minCardinality`, `maxCardinality`, `cardinality`, `someValuesFrom` with these new qualified cardinality restrictions? Explain how.

   It is possible:

$$\begin{aligned}\texttt{restriction}(p, \texttt{minCardinality}(n)) &\equiv \texttt{restriction}(p, \texttt{minQualifiedCardinality}(n, \texttt{Thing})) \\ \texttt{restriction}(p, \texttt{maxCardinality}(n)) &\equiv \texttt{restriction}(p, \texttt{maxQualifiedCardinality}(n, \texttt{Thing})) \\ \texttt{restriction}(p, \texttt{cardinality}(n)) &\equiv \texttt{restriction}(p, \texttt{qualifiedCardinality}(n, \texttt{Thing})) \\ \texttt{restriction}(p, \texttt{someValuesFrom}(C)) &\equiv \texttt{restriction}(p, \texttt{minQualifiedCardinality}(1, C))\end{aligned}$$

Consider, in addition to the previous RDF graphs, the following statements (expressed in OWL 1):

```
ex:womanmember owl:subPropertyOf ex:member .
ex:womanmember rdfs:range ex:Woman .
ex:ModernTeam1 rdfs:subClassOf ex:SmallTeam .
ex:ModernTeam1 rdfs:subClassOf _:b .
_:b rdf:type owl:Restriction .
_:b owl:onProperty ex:womanmember .
_:b owl:minCardinality 4 .
```

3. Does ex:ModernTeam1 subsume ex:ModernTeam2 or the other way around? Justify.

   It is clear that any ex:ModernTeam1 is a ex:ModernTeam2 because the ex:womanmember are ex:members who are ex:Woman, so having 4 of them will satisfy the required constraint in ex:ModernTeam2. Hence, ex:ModernTeam2 subsumes ex:ModernTeam1.

   The other way around is less clear a priori. However, when one interprets a statement like:

```
<owl:ObjectProperty rdf:about="#womanmember">
  <owl:subPropertyOf rdf:about="#member" />
  <rdfs:range  rdf:about="#Woman" />
</owl:ObjectProperty>
```

   in OWL 1 semantics, this is interpreted definitionally, i.e., ex:womanmember is equivalent to those ex:member who are ex:Woman. I.e.,

$$
\begin{aligned}
& E_C(\texttt{restriction}(\texttt{ex:womanmember}, \texttt{minCardinality}(4))) \\
= & \{x \in O; |\{\langle x, y\rangle \in E_R(\texttt{ex:womanmember})| \geq 4\} \\
= & \{x \in O; |\{\langle x, y\rangle \in E_R(\texttt{ex:member} \sqcap \texttt{range}(\texttt{ex:Woman}))| \geq 4\} \\
= & \{x \in O; |\{\langle x, y\rangle \in E_R(\texttt{ex:member})\} \cap E_R(\texttt{range}(\texttt{ex:Woman}))| \geq 4\} \\
= & \{x \in O; |\{\langle x, y\rangle \in E_R(\texttt{ex:member})\} \cap \{\langle x, y\rangle; y \in E_C(\texttt{ex:Woman}))\}| \geq 4\} \\
= & \{x \in O; |\{\langle x, y\rangle \in E_R(\texttt{ex:member}); y \in E_C(\texttt{ex:Woman}))| \geq 4\} \\
= & E_C(\texttt{restriction}(\texttt{ex:member}, \texttt{minQualifiedCardinality}(4, \texttt{ex:Woman})))
\end{aligned}
$$

   Hence the two expressions are equivalent and ex:ModernTeam1 subsumes ex:ModernTeam2.

4. Does this suggest that it is also possible to express qualified cardinality constraints in OWL 1? Explain.

   This indeed suggests that, by adding property definitions, such as ex:womanmember, it is possible to transform any OWL 1 ontology with qualified cardinality restrictions into an equivalent plain OWL 1 ontology, i.e., without qualified cardinality restrictions.

5. Does qualified cardinality restrictions provide additional expressivity to OWL 1?

   Hence, no. Qualified cardinality restrictions are only syntactic sugar.